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MPI Programming

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Outline

- Introduction
- Basic MPI programming
- Examples
 - Finite Difference Method
 - Finite Element Method
 - LU Factorization
 - Monte Carlo Method
 - Molecular Dynamics
 - MPMD Models



Introduction

Acknowledgments

- Thanks to Professor Lasse Natvig and Associate Professor Jørn Amundsen from IDI, NTNU for allowing me to copy from their “Parallel Programming” lecture slides
- Thanks to IBM for allowing me to copy from their MPI redbook “RS/6000 SP: Practical MPI Programming”



The Examples

- No exercises, but 7 working examples are explained
 - The last 6 are larger examples
- On Idun or Saga:
 - cp -r /cluster/home/hrn/Kurs/mpi .
 - module load foss/2021b

Basic MPI Programming

MPI Programs in C

- A C program
 - Has a main() function
 - Includes stdio.h, string.h, etc.
- Need to include **mpi.h** header file
- Identifiers defined by MPI start with “MPI_”
- First letter following underscore is uppercase
 - For function names and MPI-defined types
 - Helps to avoid confusion

MPI Programs in Fortran

- include 'mpif.h'
 - No argument checking! Don't use it.
- use mpi
 - Provide explicit interfaces for all MPI routines → compile time argument checking
- use mpi_f08
 - Fully Fortran 2008 compatible definition of all MPI routines
 - New syntax TYPE(*), DIMENSION(...) to define choice buffers in a standardized way



Identifying MPI Processes

- Common practice is to identify processes by non-negative integer ranks
- p processes are numbered 0, 1, 2, ..., $p-1$
- This can be:
 - p processes distributed over p processors (“physical parallelism”)
 - p processes running time-multiplexed on a single processor (“logical parallelism”)



Example 1: Hello, World!

- Compile the code:

```
$ cd examples/ex1  
$ make  
mpicc -O2 mpi_hello.c -o mpi_hello
```

- Edit the job script:

```
$ vim run.slurm (change <ACCOUNT>)
```

- Run the job:

```
$ sbatch run.slurm
```



MPI Start and End

- **`MPI_Init()`**
 - Tells MPI to do all the necessary setup

```
int MPI_Init(  
    int* argc_p  
    char*** argv_p);
```

Pointers to the arguments
to main: argc & argv

- **`MPI_Finalize()`**
 - Tells MPI we're done, so clean up anything allocated

```
int MPI_Finalize(void);
```



Basic Outline

```
...
#include <mpi.h>
...
int main(int argc, char* argv[]) {
    ...
    /* No MPI calls before this */
    MPI_Init(&argc, &argv);
    ...
    MPI_Finalize();
    /* No MPI calls after this */
    ...
    return 0;
}
```



Communicators

- A collection of processes that can send messages to each other
- **MPI_Init()** defines a communicator that consists of all the processes created when the program is started
 - **MPI_COMM_WORLD**

Communicators

```
int MPI_Comm_size(  
    MPI_Comm     comm /* in */,  
    int*         size /* out */);
```

number of processes in the communicator

```
int MPI_Comm_rank(  
    MPI_Comm     comm /* in *//  
    int*         rank /* out */);
```

my rank

(the process making this call)



Communication

```
int MPI_Send(  
    void*           buf /* in */,  
    int             count /* in */,  
    MPI_Datatype   datatype /* in */,  
    int             dest /* in */,  
    int             tag /* in */,  
    MPI_Comm       communicator /* in */);
```

number of elements in
the send buffer

Data Types

MPI datatype	C datatype
<code>MPI_CHAR</code>	<code>signed char</code>
<code>MPI_SHORT</code>	<code>signed short int</code>
<code>MPI_INT</code>	<code>signed int</code>
<code>MPI_LONG</code>	<code>signed long int</code>
<code>MPI_LONG_LONG</code>	<code>signed long long int</code>
<code>MPI_UNSIGNED_CHAR</code>	<code>unsigned char</code>
<code>MPI_UNSIGNED_SHORT</code>	<code>unsigned short int</code>
<code>MPI_UNSIGNED</code>	<code>unsigned int</code>
<code>MPI_UNSIGNED_LONG</code>	<code>unsigned long int</code>
<code>MPI_FLOAT</code>	<code>float</code>
<code>MPI_DOUBLE</code>	<code>double</code>
<code>MPI_LONG_DOUBLE</code>	<code>long double</code>
<code>MPI_BYTE</code>	
<code>MPI_PACKED</code>	



Communication

```
int MPI_Recv(  
    void*           buf,  
    int             count, /* out */,  
    MPI_Datatype   datatype, /* in */,  
    int             source, /* in */,  
    int             tag, /* in */,  
    MPI_Comm        communicator, /* in */,  
    MPI_Status*     status); /* out */;
```

max. number of elements
to receive

Message Matching

- Process **q** calls **MPI_Send()**

```
MPI_Send(send_buf, send_count, send_type, dest,  
         send_tag, send_comm);
```

- Process **r** calls **MPI_Recv()**

```
MPI_Recv(recv_buf, recv_count, recv_type, src,  
         recv_tag, recv_comm, &status);
```



Receiving Messages

- A receiver can get a message without knowing:
 - the amount of data in the message,
 - the sender of the message
 - MPI_ANY_SOURCE
 - or the tag of the message
 - MPI_ANY_TAG

The Status Argument

- If `MPI_ANY_SOURCE` or `MPI_ANY_TAG` have been used, you can get help from `MPI_Status`

```
MPI_Recv(buf, count, datatype, src  
         tag, comm, &status) ;
```

```
MPI_Status status;  
  
status.MPI_SOURCE  
status.MPI_TAG
```



How much data am I receiving?

```
int MPI_Probe(  
    int source /* in */,  
    int tag /* in */,  
    MPI_Comm comm /* in */  
    MPI_Status* status /* out */);  
  
int MPI_Get_count(  
    MPI_Status* status /* in */,  
    MPI_Datatype datatype /* in */,  
    int* count /* out */);
```



Issues with Send and Receive

- Exact behavior is determined by the MPI implementation
- `MPI_Send()` is blocking as defined in the standard, but is non-blocking up to a certain message size in most implementations
 - `MPI_Ssend()` might be used to force blocking until a receive is posted
 - `MPI_Bsend()` can be used with a user defined send buffer - then always non-blocking
- `MPI_Recv()` always blocks until a matching message is received

Issues with Send and Receive

- AND, ...
 - MPI programs will easily hang!
 - A receive without corresponding send
 - A send without corresponding receive
 - Or deadlock
 - Circular waiting



Non-Blocking Communication

```
int MPI_Isend(void* buffer, int count, MPI_Datatype  
datatype, int destination, int tag, MPI_Comm comm,  
MPI_Request* request);  
  
int MPI_Irecv(void* buffer, int count, MPI_Datatype  
datatype, int source, int tag, MPI_Comm comm,  
MPI_Request* request);  
  
int MPI_Wait(MPI_Request* request, MPI_Status* status);  
  
int MPI_Waitall(int array_size,  
MPI_Request requests[], MPI_Status statuses[]);
```

I = Immediate



Non-Blocking Communication

- Immediate-mode `MPI_Isend()` and `MPI_Irecv()` only start the data copy operation
- `MPI_Wait()` and `MPI_Waitall()` are used to complete the operations
- Useful in complicated send-receive situations (e.g. 2D grid of processes)
- Calculations can take place between those two calls
 - Difficult to make good use of
 - Communication and calculation at the same time is more efficient



Global Reduction

```
int MPI_Reduce(void *sendbuf, void *recvbuf, int count  
               MPI_Datatype datatype, MPI_Op op, int root,  
               MPI_Comm comm);
```

- **op** determines which global reduction to perform
- Predefined reductions for the most used types, like **MPI_MAX**, **MPI_MIN**, **MPI_SUM**, **MPI_PROD**, etc.
- Also possible to specify user defined reduction operations with **MPI_Op_create()**
- **MPI_IN_PLACE** specified for sendbuf at rank **root**, makes the receive buffer a send-and-receive buffer



Broadcast

```
int MPI_Bcast(void *buffer, int count, MPI_Datatype  
Datatype, int root, MPI_Comm comm);
```

- Broadcasts a message from the process with rank **root** to all other processes of the communicator



Scatter and gather

```
int MPI_Scatter(void *sendbuf, int sendcount, MPI_Datatype  
    sendtype, void *recvbuf, int recvcount,  
    MPI_Datatype recvtype, int root, MPI_Comm comm);
```

```
int MPI_Gather(void *sendbuf, int sendcount, MPI_Datatype  
    sendtype, void *recvbuf, int recvcount,  
    MPI_Datatype recvtype, int root, MPI_Comm comm);
```

- Routines to spread and collect data from or to **root**
- **MPI_Scatter**: if **root** sends 100 numbers to 10 processes, then **sendbuf** on **root** must be 1000 long
- **MPI_Gather**: if **root** receives 100 numbers from 10 processes, then **recvbuf** on **root** must be 1000 long



Allreduce, allgather and more

- **MPI_Allreduce()** and **MPI_Allgather()** are identical to their siblings, except that the end result is made available to all ranks
- As if the operation was followed by a broadcast
- There are also more elaborate combined all-to-all scatter-gather functions, like **MPI_Alltoall()**, **MPI_Alltoallv()** and **MPI_Alltoallw()**
- Use the man pages to get more information, e.g.:
`$ man MPI_Allreduce`



Examples



Writing Larger MPI Programs

- Question:
 - Now that we can write “Hello World!” MPI programs, then what do we need in order to write larger MPI programs for scientific projects?
- Answer:
 - Parallel algorithms
 - Data must be distributed to all processes so that they all are kept busy during the entire execution of MPI programs



Data Distribution

- The majority of time is usually spent in DO/FOR loops
- Multiple data distribution methods:
 - Block distribution
 - Column wise
 - Row wise
 - In both dimensions
 - Cyclic distribution
 - Master-worker
- 6 examples



2. Finite Difference Method

- Wikipedia:
“Numerical methods for solving differential equations by approximating them with difference equations”
- Only a skeleton 2D FDM program is shown here
- Coefficients and the enclosing loop are omitted
- Data dependencies exist in both dimensions

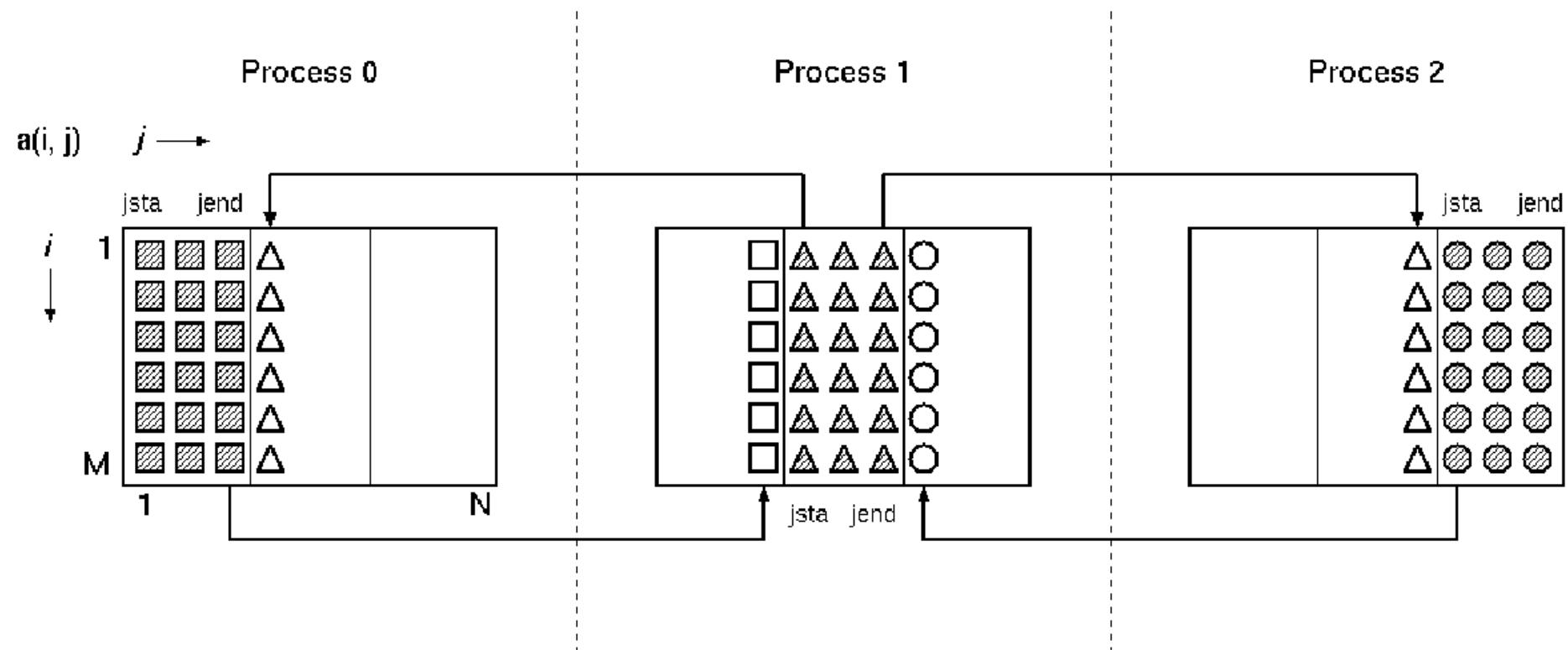


The Sequential Algorithm

```
PROGRAM main
IMPLICIT REAL*8 (a-h,o-z)
PARAMETER (m=6, n=9)
DIMENSION a(m,n), b(m,n)
DO j=1, n
    DO i=1, m
        a(i,j) = i + 10.0 * j
    ENDDO
ENDDO
DO j=2, n-1
    DO i=2, m-1
        b(i,j) = a(i-1,j) + a(i,j-1) + a(i,j+1) + a(i+1,j)
    ENDDO
ENDDO
END
```



Column-Wise Block Distribution



Column-Wise Block Distribution

- We must distribute a 2D matrix onto the processes
- Fortran stores arrays in column-major order
- Boundary elements between processes are contiguous in memory
- There are no problems with using MPI_SEND and MPI_RECV



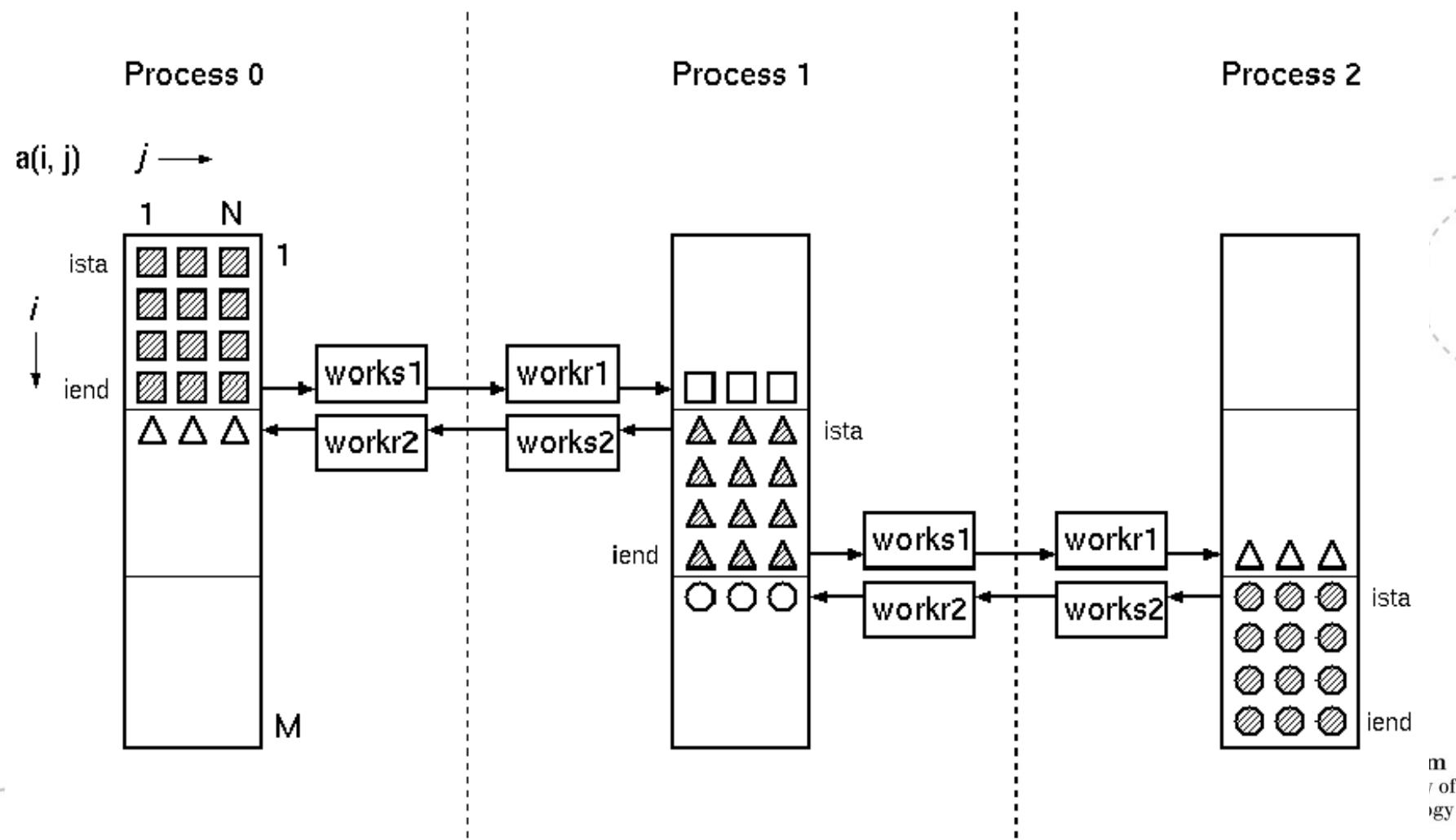
Example 2

ex2/fdm1.f90



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Row-Wise Block Distribution



Row-Wise Block Distribution

- Fortran stores arrays in column-major order
- Boundary elements between processes are not contiguous in memory
- Boundary elements can be copied by:
 - Using derived data types
 - Writing code for packing data, sending/receiving it, and then unpacking it



Example 2

ex2/fdm2.f90



Block Distribution in Both Dim. (1)

- The amount of data transferred might be minimized
 - Depends upon the matrix size and the number of processes
- A process grid table is prepared for looking up processes quickly



Block Distribution in Both Dim. (1)

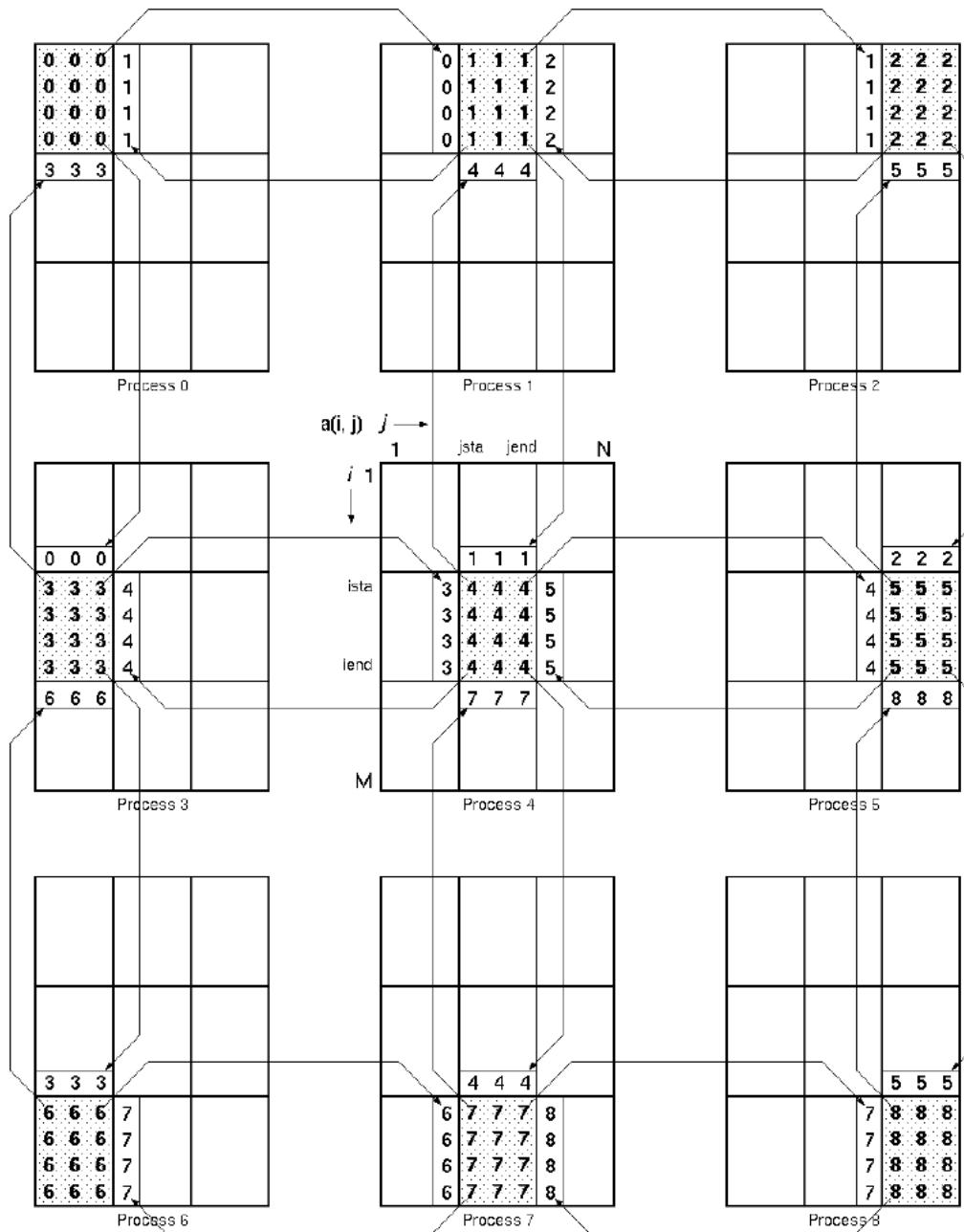
$a(i, j) \quad j \rightarrow$

		1		N
i	1	0 0 0	1 1 1	2 2 2
	↓	0 0 0	1 1 1	2 2 2
		0 0 0	1 1 1	2 2 2
		0 0 0	1 1 1	2 2 2
		3 3 3	4 4 4	5 5 5
		3 3 3	4 4 4	5 5 5
		3 3 3	4 4 4	5 5 5
		3 3 3	4 4 4	5 5 5
		6 6 6	7 7 7	8 8 8
		6 6 6	7 7 7	8 8 8
		6 6 6	7 7 7	8 8 8
M		6 6 6	7 7 7	8 8 8

(a) The distribution of $a()$

		$j \rightarrow$			
itable(i, j)	-1	0	1	2	3
i	-1	null	null	null	null
	0	0	1	2	null
	1	3	4	5	null
	2	6	7	8	null
	3	null	null	null	null

(b) The process grid



Example 2

ex2/fdm3.f90



Block Distribution in Both Dim. (2)

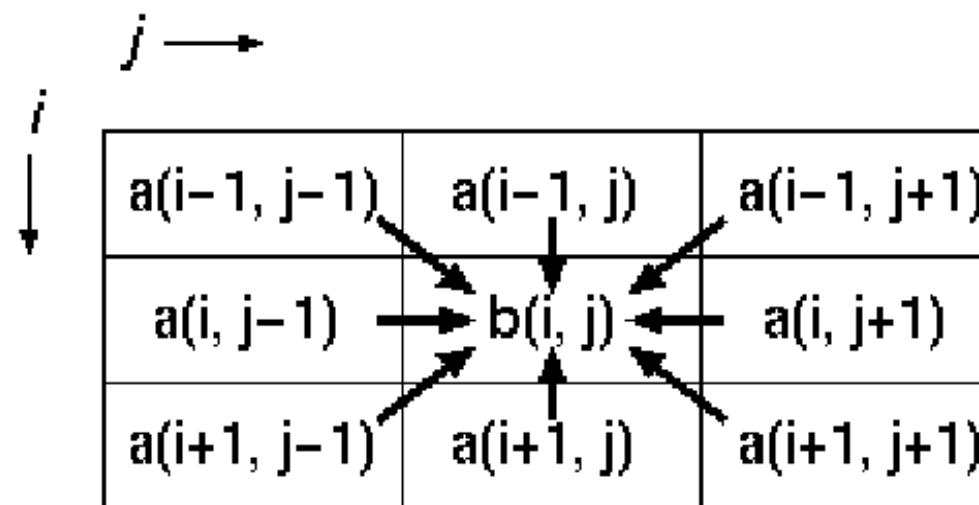
- The corner elements are now included
- The data dependencies are therefore more complex

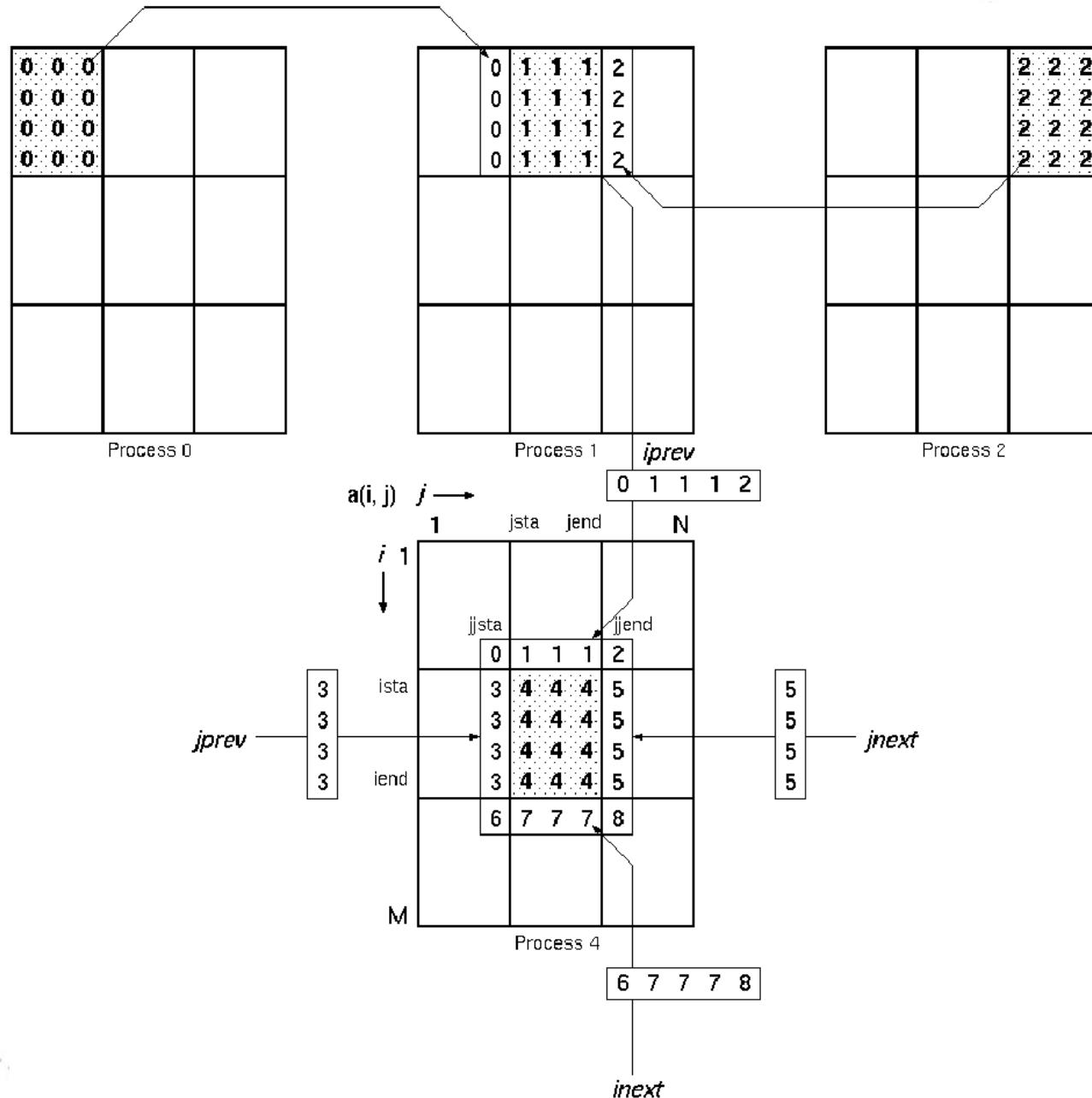
The Sequential Algorithm

```
PROGRAM main
IMPLICIT REAL*8 (a-h,o-z)
PARAMETER (m=12, n=9)
DIMENSION a(m,n), b(m,n)
DO j=1, n
    DO i=1, m
        a(i,j) = i + 10.0 * j
    ENDDO
ENDDO
DO j=2, n-1
    DO i=2, m-1
        b(i,j) = a(i-1,j) + a(i,j-1) + a(i,j+1) + a(i+1,j) + &
                   a(i-1,j-1) + a(i+1,j-1) + a(i-1,j+1) +a(i+1,j+1)
    ENDDO
ENDDO
END
```



The Data Dependency





Example 2

ex2/fdm4.f90



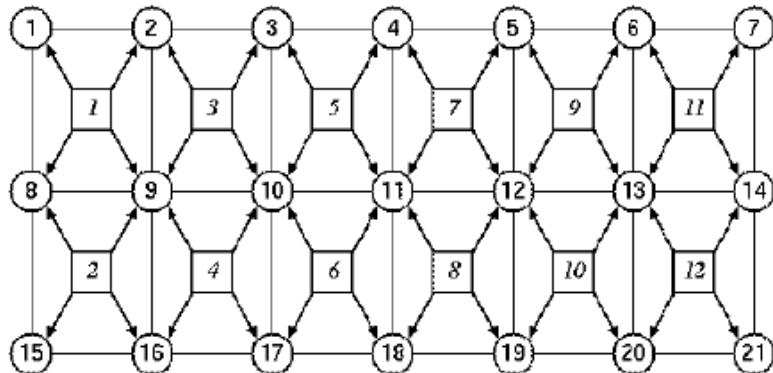
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3. Finite Element Method

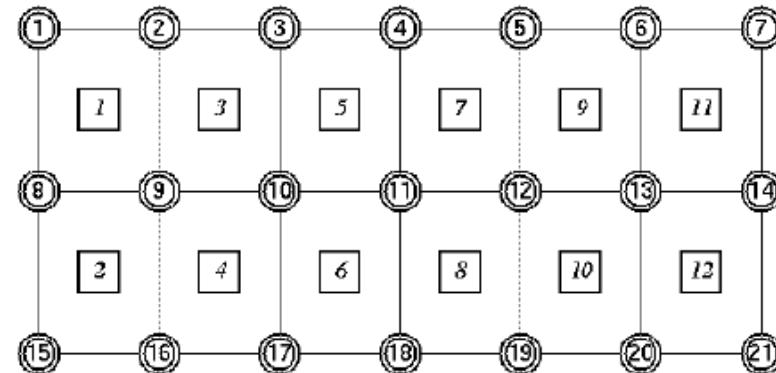
- Wikipedia:
“Numerical technique for finding approximate solutions to boundary value problems for partial differential equations”
- A more complete example that produces a result



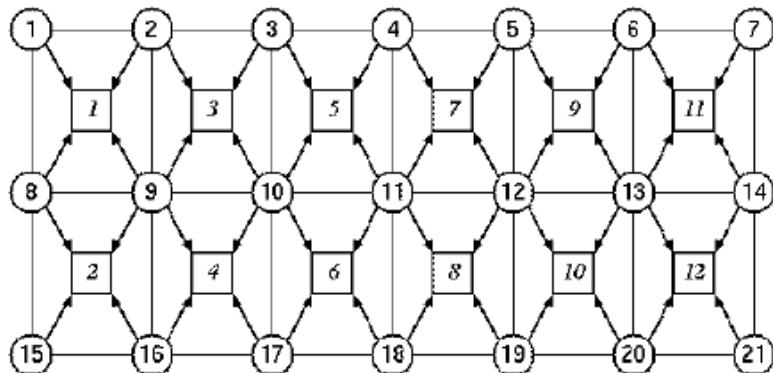
Finite Element Method



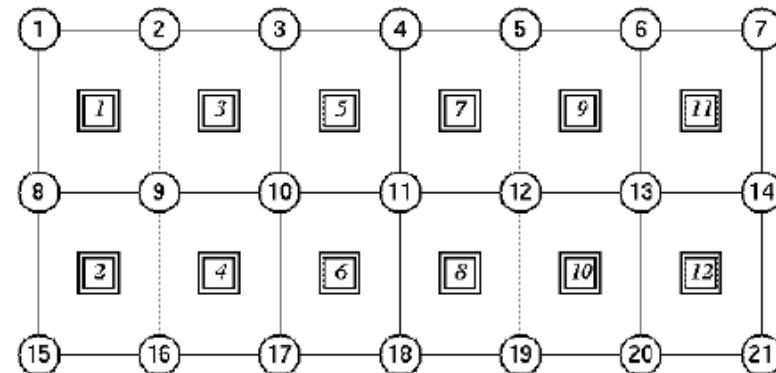
(a) Elements → Nodes



(b) Update Nodes



(c) Nodes → Elements



(d) Update Elements

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The Sequential Algorithm

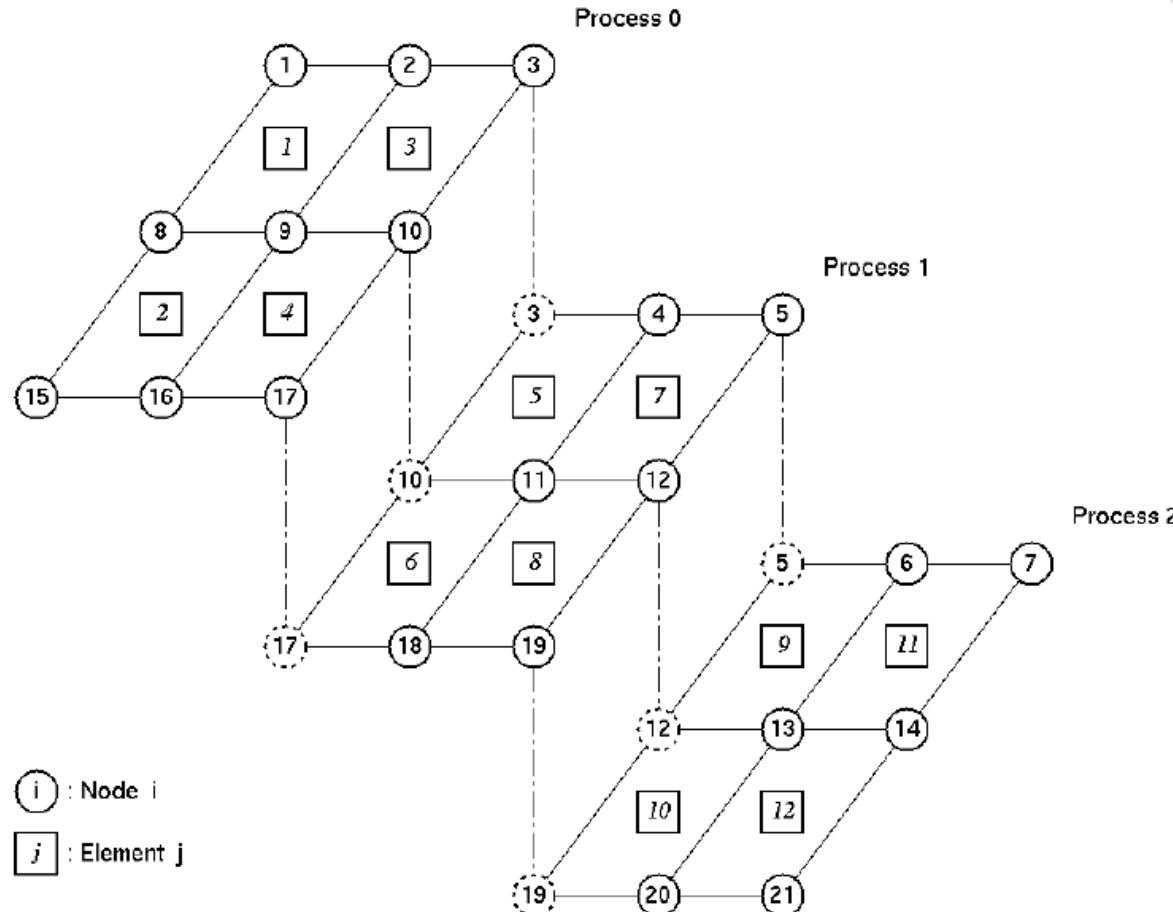
```
PARAMETER(iemax=12, inmax=21)
REAL*8 ve(iemax), vn(inmax)
INTEGER index(4, iemax)
...
DO ie=1, iemax
    ve(ie) = ie * 10.0
ENDDO
DO in=1, inmax
    vn(in) = in * 100.0
ENDDO
DO itime=1, 10
    DO ie=1, iemax
        DO j=1, 4
            vn(index(j,ie)) =
vn(index(j,ie)) + ve(ie)
        ENDDO
    ENDDO

```

```
    DO in = 1, inmax
        vn(in) = vn(in) * 0.25
    ENDDO
    DO ie = 1, iemax
        DO j = 1, 4
            ve(ie) = ve(ie) + vn(index(j,ie))
        ENDDO
    ENDDO
    DO ie = 1, iemax
        ve(ie) = ve(ie) * 0.25
    ENDDO
PRINT *, 'Result',vn,ve
```



Distributing the Data



Differences from IBM version

- 2D enumeration (row, column) is used instead of 1D enumeration
- The amount of memory allocated by each process is minimized
- A node column is sent to the right
- An element column is sent to the left



Example 3

ex3/main.f90 and ex3/grid.f90



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4. LU Factorization

- Wikipedia:
 - “Factors a matrix as the product of a lower triangular matrix and an upper triangular matrix”
 - Used for solving square systems of linear equations:
 $Ax = b$
- ScaLAPACK and Intel's MKL library have optimized subroutines for this (outside the scope of this course)
- Pivoting and loop-unrolling is not considered



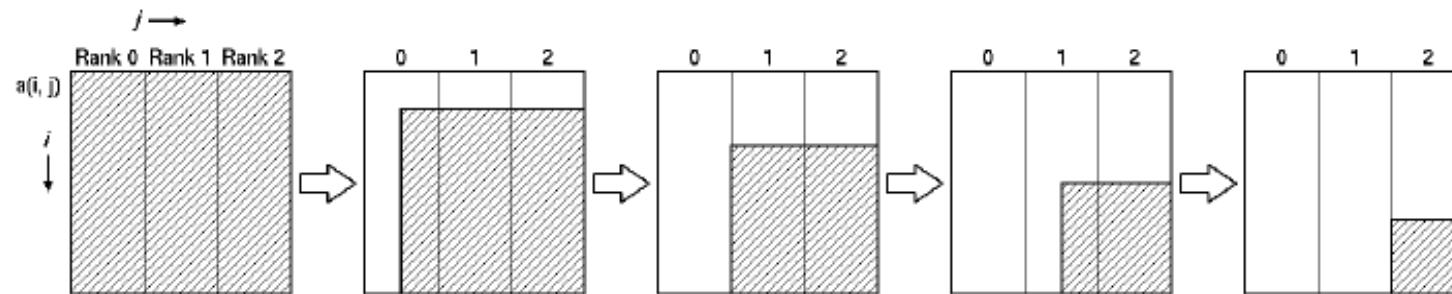
The Sequential Algorithm

```
PROGRAM main
PARAMETER (n = ...)
REAL a(n,n), b(n)
...
! LU factorization
DO k = 1, n-1
    DO i = k+1, n
        a(i,k) = a(i,k) / a(k,k)
    ENDDO
    DO j = k+1, n
        DO i = k+1, n
            a(i,j) = a(i,j)-a(i,k)*a(k,j)
        ENDDO
    ENDDO
ENDDO
ENDDO
```

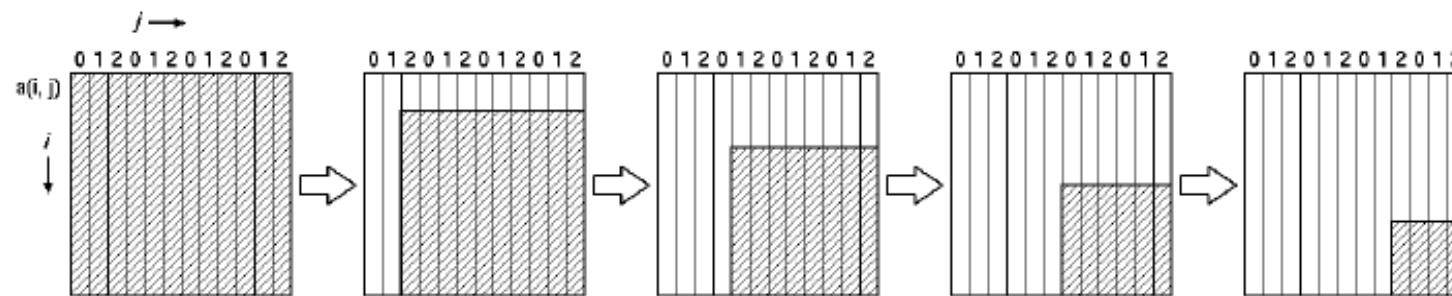
```
! Forward elimination
DO i = 2, n
    DO j = 1, i - 1
        b(i) = b(i) - a(i,j)*b(j)
    ENDDO
ENDDO
! Backward substitution
DO i = n, 1, -1
    DO j = i + 1, n
        b(i) = b(i) - a(i,j)*b(j)
    ENDDO
    b(i) = b(i) / a(i,i)
ENDDO
...
END
```

Cyclic Data Distributing

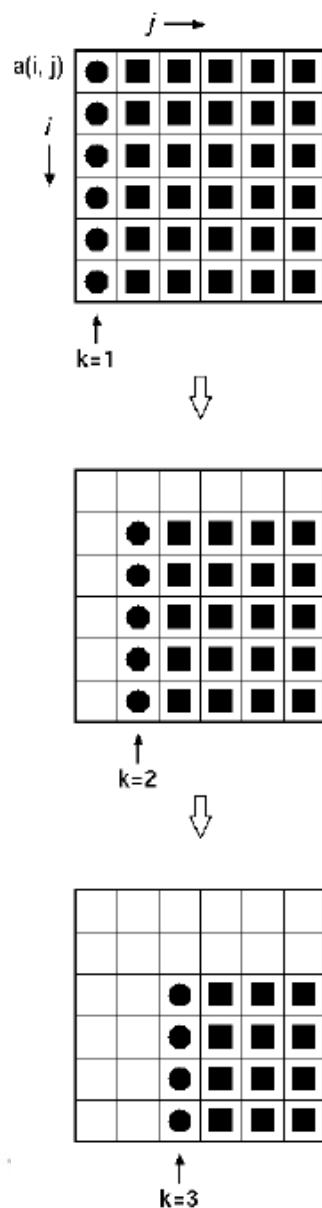
(a) Block distribution



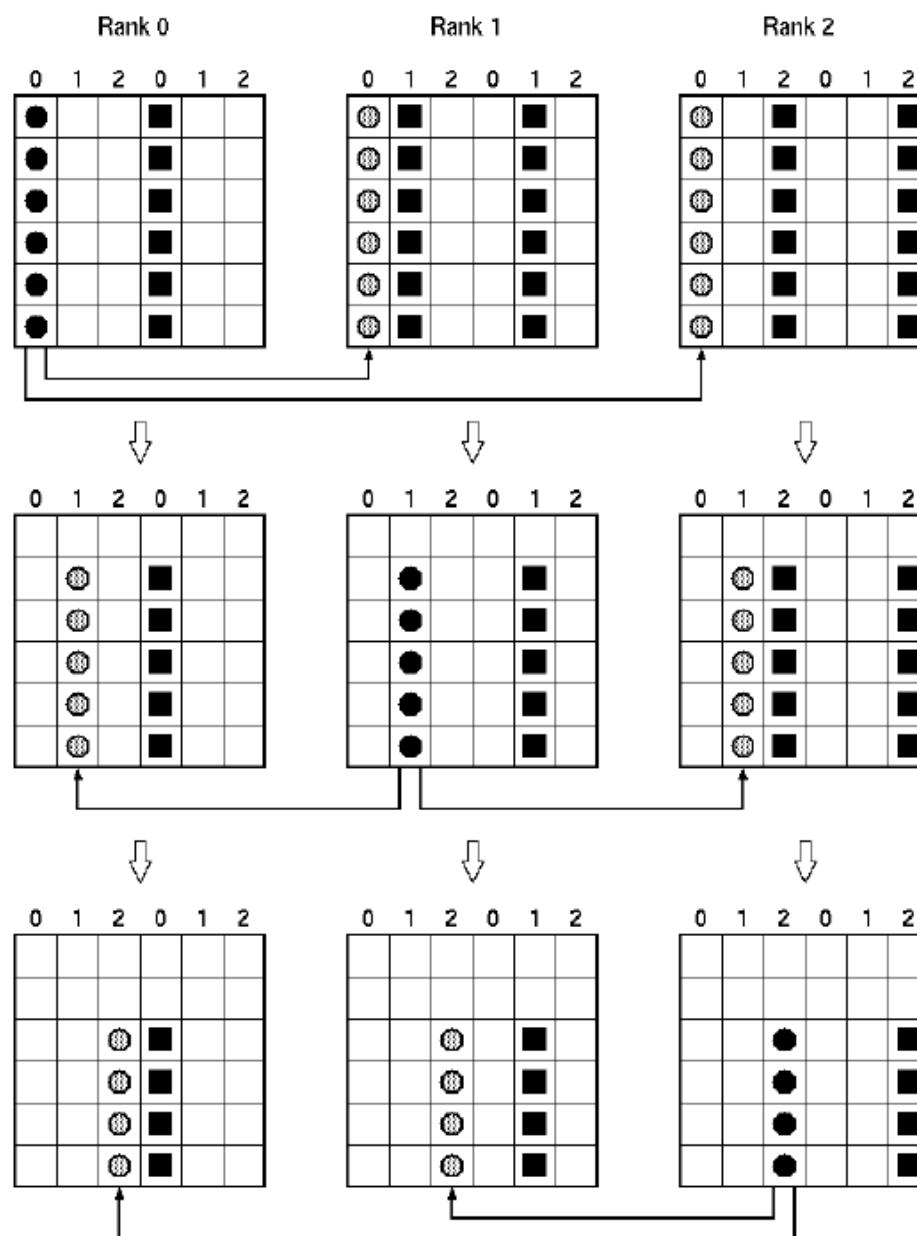
(b) Cyclic distribution



Serial execution



Parallel execution



Example 4

ex4/lu.f90

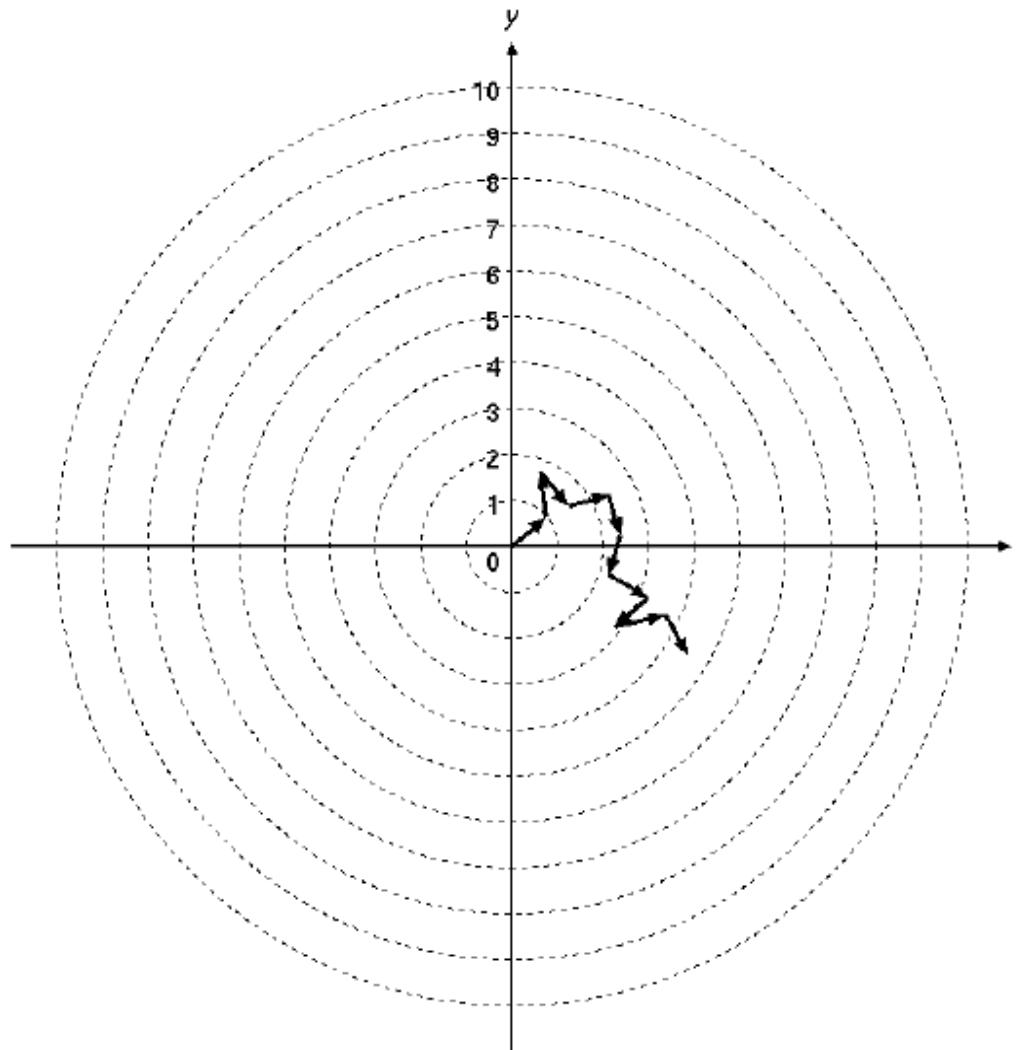


5. The Monte Carlo Method

- Wikipedia:
“A broad class of computational algorithms that rely on repeated random sampling to obtain numerical results”
- A random walk in 2D
- 100,000 particles
- 10 steps



A Sample Trajectory



The Sequential Algorithm

```
PROGRAM main
PARAMETER (n=100000)
INTEGER itotal(0:9)
REAL seed
pi = 3.1415926
DO i = 0, 9
    itotal(i) = 0
ENDDO
seed = 0.5
CALL srand(seed)
```

```
DO i = 1, n
    x = 0.0
    y = 0.0
    DO istep = 1, 10
        angle = 2.0 * pi * rand()
        x = x + cos(angle)
        y = y + sin(angle)
    ENDDO
    itemp = sqrt(x**2 + y**2)
    itotal(itemp) = itotal(itemp) + 1
ENDDO
PRINT *, 'total =' ,itotal
END
```

Example 5

ex5/mc.f90

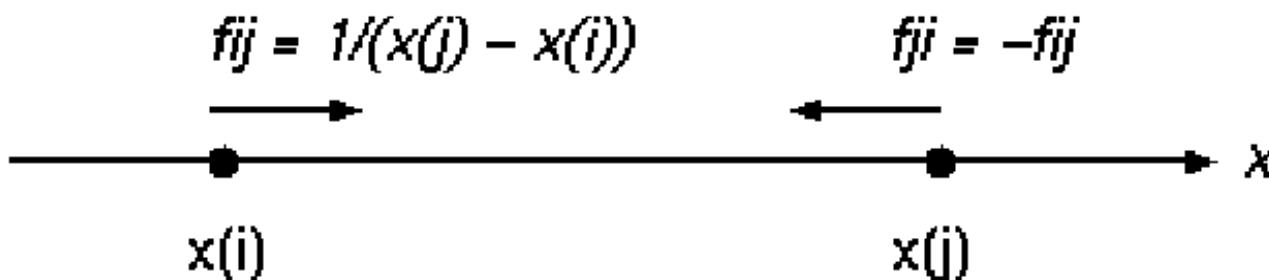


6. Molecular Dynamics

- Wikipedia:
“a computer simulation of physical movements of atoms and molecules”
- N particles interact in 1 dimension
- The force on particle i from particle j is given by
$$f_{ij} = 1/(x_j - x_i)$$
- The law of action and reaction applies:
$$f_{ij} = -f_{ji}$$



Forces in 1D



Forces Acting on 7 Particles

$f(1) =$		+f12	+f13	+f14	+f15	+f16	+f17
$f(2) =$	-f12		+f23	+f24	+f25	+f26	+f27
$f(3) =$	-f13	-f23		+f34	+f35	+f36	+f37
$f(4) =$	-f14	-f24	-f34		+f45	+f46	+f47
$f(5) =$	-f15	-f25	-f35	-f45		+f56	+f57
$f(6) =$	-f16	-f26	-f36	-f46	-f56		+f67
$f(7) =$	-f17	-f27	-f37	-f47	-f57	-f67	



The Sequential Algorithm

```
PARAMETER (n = . . .)
REAL f(n), x(n)

...
DO itime = 1, 100
  DO i = 1, n
    f(i) = 0.0
  ENDDO
  DO i = 1, n-1
    DO j = i+1, n
      fij = 1.0 / (x(j)-x(i))
      f(i) = f(i) + fij
      f(j) = f(j) - fij
    ENDDO
  ENDDO
  DO i = 1, n
    x(i) = x(i) + f(i)
  ENDDO
ENDDO
```



Two Parallelisation Methods

- Most of the time is spent in the calculation loop:

```
DO i = 1, n-1
    DO j = i+1, n
        fij = 1.0/(x(j) - x(i))
        f(i) = f(i) + fij
        f(j) = f(j) - fij
    ENDDO
ENDDO
```

- Two parallelization methods:
 - Cyclic distribution of the outer loop
 - Cyclic distribution of the inner loop



Process 0

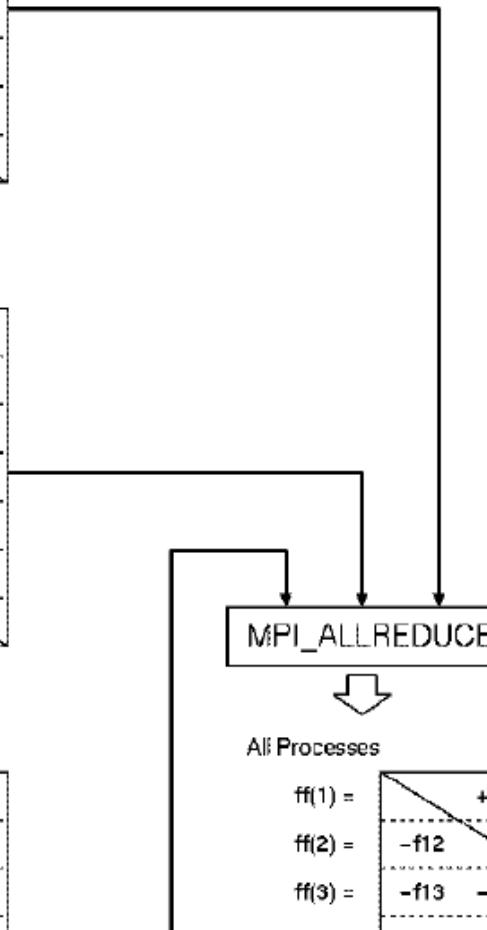
$f(1) =$	+f12	+f13	+f14	+f15	+f16	+f17
$f(2) =$	-f12					
$f(3) =$	-f13					
$f(4) =$	-f14		+f45	+f46	+f47	
$f(5) =$	-f15		-f45			
$f(6) =$	-f16		-f46			
$f(7) =$	-f17		-f47			

Process 1

$f(1) =$						
$f(2) =$		+f23	+f24	+f25	+f26	+f27
$f(3) =$	-f23					
$f(4) =$	-f24					
$f(5) =$	-f25		+f56	+f57		
$f(6) =$	-f26		-f56			
$f(7) =$	-f27		-f57			

Process 2

$f(1) =$						
$f(2) =$						
$f(3) =$		+f34	+f35	+f36	+f37	
$f(4) =$	-f34					
$f(5) =$	-f35					
$f(6) =$	-f36					
$f(7) =$	-f37		-f67			



All Processes

$ff(1) =$	+f12	+f13	+f14	+f15	+f16	+f17
$ff(2) =$	-f12	+f23	+f24	+f25	+f26	+f27
$ff(3) =$	-f13	-f23	+f34	+f35	+f36	+f37
$ff(4) =$	-f14	-f24	-f34	+f45	+f46	+f47
$ff(5) =$	-f15	-f25	-f35	-f45	+f56	+f57
$ff(6) =$	-f16	-f26	-f36	-f46	-f56	+f67
$ff(7) =$	-f17	-f27	-f37	-f47	-f57	-f67

Example 6

ex6/md1.f90



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Process 0

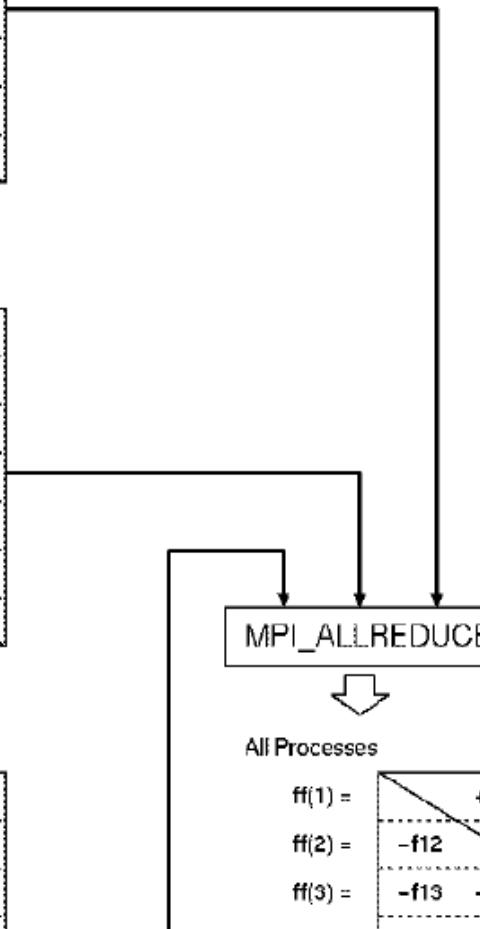
f(1) =	+f12	+f15				
f(2) =	-f12	+f23	+f26			
f(3) =	-f23	+f35				
f(4) =		+f45				
f(5) =	-f15	-f35	-f45	+f56		
f(6) =	-f26		-f56			
f(7) =						

Process 1

f(1) =	+f13	+f16				
f(2) =		+f24	+f27			
f(3) =	-f13		+f35			
f(4) =	-f24		+f46			
f(5) =			+f57			
f(6) =	-f16	-f36	-f46			
f(7) =	-f27		-f57			

Process 2

f(1) =	+f14	+f17				
f(2) =		+f25				
f(3) =		+f34	+f37			
f(4) =	-f14	-f34				
f(5) =	-f25		+f47			
f(6) =	-f17	-f37	-f47	-f67		
f(7) =						



All Processes

ff(1) =	+f12	+f13	+f14	+f15	+f16	+f17
ff(2) =	-f12	+f23	+f24	+f25	+f26	+f27
ff(3) =	-f13	-f23	+f34	+f35	+f36	+f37
ff(4) =	-f14	-f24	-f34	+f45	+f46	+f47
ff(5) =	-f15	-f25	-f35	-f45	+f56	+f57
ff(6) =	-f16	-f26	-f36	-f46	-f56	+f67
ff(7) =	-f17	-f27	-f37	-f47	-f57	-f67

Example 6

ex6/md2.f90



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7. MPMD Models

- Wikipedia:
“Multiple Program, Multiple Data: multiple autonomous processors simultaneously operating at least 2 independent programs”
- Different programs run in parallel and communicate with each other



An example

Process 0

```
PROGRAM fluid
INCLUDE 'mpif.h'
...
CALL MPI_INIT
CALL MPI_COMM_SIZE
CALL MPI_COMM_RANK
...
DO itime = 1, n
    Computation of
    Fluid Dynamics
    CALL MPI_SEND
    CALL MPI_RECV
ENDDO
...
END
```

Process 1

```
PROGRAM struct
INCLUDE 'mpif.h'
...
CALL MPI_INIT
CALL MPI_COMM_SIZE
CALL MPI_COMM_RANK
...
DO itime = 1, n
    Computation of
    Structural Analysis
    CALL MPI_RECV
    CALL MPI_SEND
ENDDO
...
END
```

Master/Worker Programs

- The master coordinates the execution of all the other processes
- The master has a list of jobs that must be processed
- Suitable if:
 - The processing time varies greatly from job to job
 - Neither block nor cyclic distribution gives a good load balancing
 - A heterogeneous environment where the performance of the machines is not uniform



Example 7

ex7/master.f90 and ex7/worker.f90



More Information

- Examples are based on:
 - IBM Redbook: “RS/6000 SP: Practical MPI Programming”
- Documentation:
 - <https://www.sigma2.no/documentation>
 - <http://www.hpc.ntnu.no/>

